NATIONAL RECONNAISSANCE OFFICE

Weibull Analysis Method

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SUPRA ET ULTRA

Erik Burgess, Burgess Consulting
James Smirnoff, Wyle
Brianne Wong, Booz Allen Hamilton

Topics

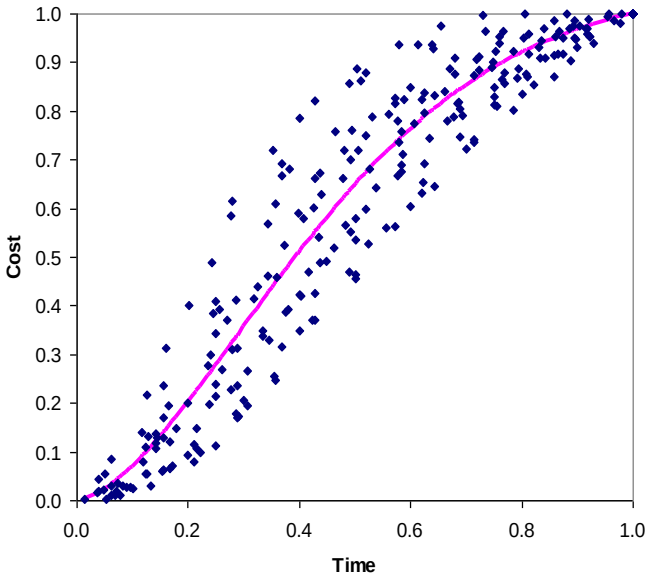
- Analytical Basis
- Accuracy
- Application

Weibull Analysis Method (WAM)

- Uses a program's actual history to estimate future budgets
  - Expenditures
  - Outlay rates
  - Government liability
- Improves accuracy over NRO's baseline parametric phasing model
  - For programs already underway
  - Especially in the near term
    - 2-years out (budget year)
    - FYDP
- Quantifies and reports error bounds based on historical data
  - Annual error vs. historical data
  - Departure from baseline model
- Builds on LMI[1] and CNA[2] research

[1] Dukovich, John et al., "The Rayleigh Analyzer." Logistics Management Institute AT902C1. October, 1999.
[2] Davis, Dan et al. "Using the Rayleigh Model to Assess Future Acquisition Contract Performance and Overall Contract Risk." Center for Naval Analysis CRM D0019289.A2, January 2009.

NRO's Baseline Parametric Phasing Model[3]



Weibull plus a constant-rate term
38 NRO & DoD Programs
387 time-cost pooled data points

$$E(t) = d\left[\, Rt + 1 - e^{-\alpha\, t^{\beta}} \,\right]$$

$$d = \frac{\text{total cost}}{R + 1\text{-}e^{-\alpha}}$$

$$0 \le t \le 1.0$$

$$\text{R} = .002945 \cdot \text{ duration (mos.)}$$

$$\alpha = 0.10 + \sum X_i \cdot \text{driver}_i$$

$$\beta = 1.539 + \sum Y_i \cdot \text{driver}_i$$

- Adjusts front/back-loading based on "phasing drivers"
- Starting point for all space-segment estimates
- Phases expenditures, converts to budget authority

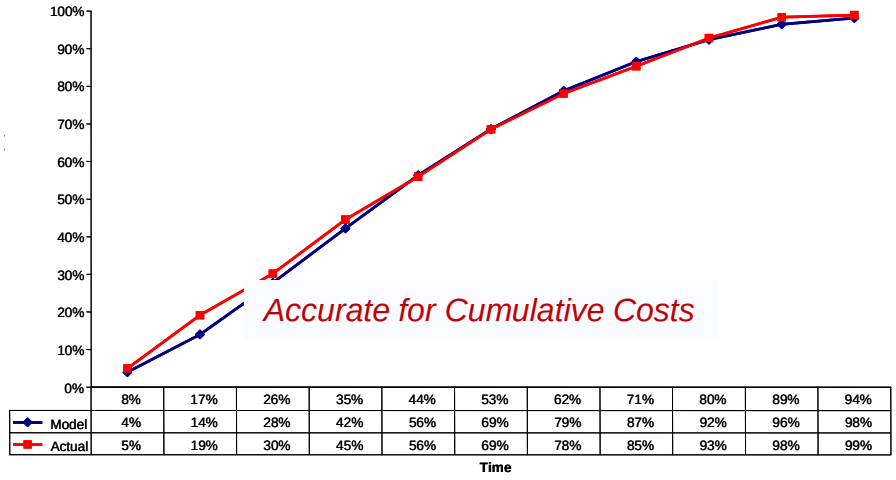
Driver	Coefficient (X)
GFE (1,0)	1.84E+00
% Subs	2.73E-02
BY07$M	9.57E-04
Duration (mos)	2.79E-02

Driver	Coefficient (Y)
Competitive (1,0)	1.71E-01
GFE (1,0)	3.62E-01
% Subs	4.47E-03
BY07$M	7.03E-05
Duration (mos)	-1.62E-03

[3] Burgess, Erik. "Modeling R&D Budget Profiles," presented at SCEA/ISPA Joint Annual Conference, Orlando, FL. June 2012.

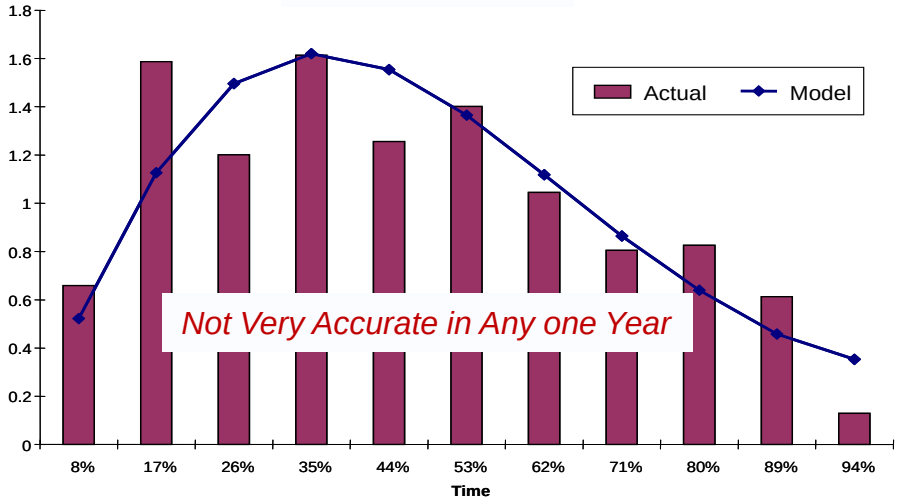
Baseline Model: Not Very Accurate in Any one Year

CUMULATIVE COSTS

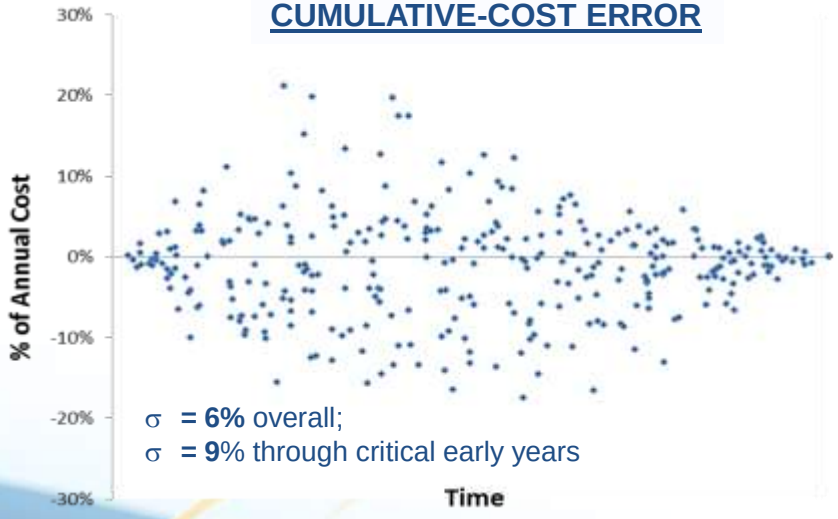


	8%	17%	26%	35%	44%	53%	62%	71%	80%	89%	94%
Model	4%	14%	28%	42%	56%	69%	79%	87%	92%	96%	98%
Actual	5%	19%	30%	45%	56%	69%	78%	85%	93%	98%	99%

ANNUAL COSTS

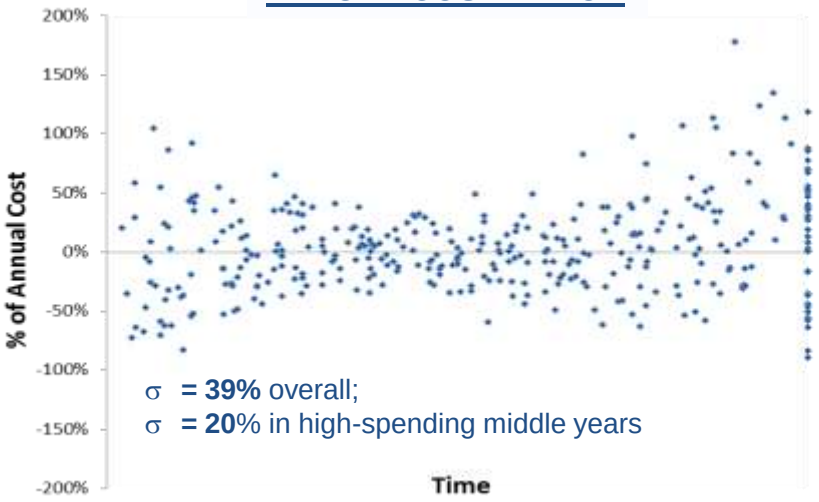


CUMULATIVE-COST ERROR



- σ = **6%** overall;
- σ = **9**% through critical early years

ANNUAL-COST ERROR



- σ = **39%** overall;
- σ = **20**% in high-spending middle years

Motivation for WAM

Baseline model establishes solid historical reference

- Cumulative accuracy through early years is quantified
- Powerful tool to link budget profile to schedule

But …

- Mid and late-program assessments now occur every year
  - Comparing government estimate at complete (GEAC) to program-office plan
  - Search for margin
  - Re-phasing the ICE
- Better method needed for evaluating annual budgets
  - Baseline model not very accurate for annual costs, especially in later years
  - Unclear how to apply baseline model when prior-year actuals are different
  - Need a method based on actuals, not plans

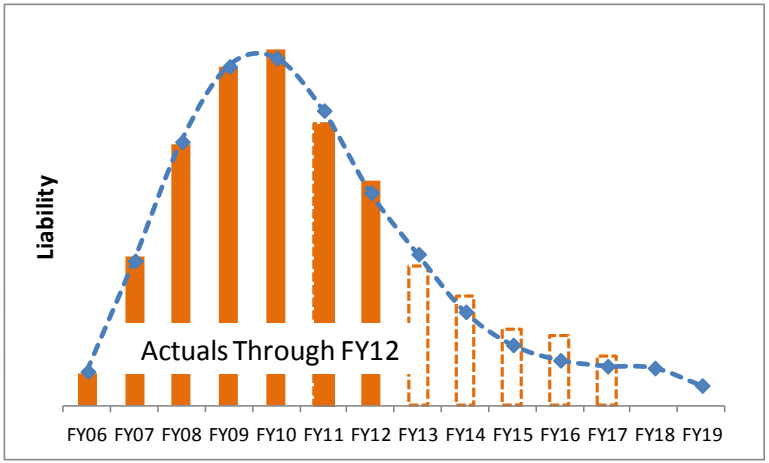
WAM: Analytical Basis

- Functional form: Weibull plus constant-rate term
  - Same as baseline phasing model
  - Empirical and theoretical basis for satellite acquisitions[4]

$$E(t) = d\left[\, Rt + 1 - e^{-\alpha\, t^{\beta}} \,\right]$$

$$d = \frac{\text{total cost}}{R + 1\text{-}e^{-\alpha}}, 0 \le t \le 1$$

- Use actual program performance to estimate Weibull parameters



[4] Burgess, Erik. "R&D Budget Profiles and Metrics". *Journal of Parametrics*. Volume VVX, Issue No. 1, Summer 2006.

Basic WAM Process

- Input: Actual expenditures for each year to date, BY$

- Constraints:
  - Total cost in BY$ (set to match ICE)
  - Schedule (set to match ICE)
  - Cumulative expenditures to date
  - Constant-rate term from baseline model: *R* = .002945 * duration

Used for re-phasing existing ICE.
Unconstrained version also can be run.

- Optimization:
  - For each year *i* of actual data: $E(t_i) = d\left[Rt_i + 1 - e^{-\alpha t_i^{\beta}}\right], d = \frac{\text{total cost}}{R + 1\text{-e}^{-\alpha}}$
  - Excel Solver© estimates Weibull parameters α, β by minimization:

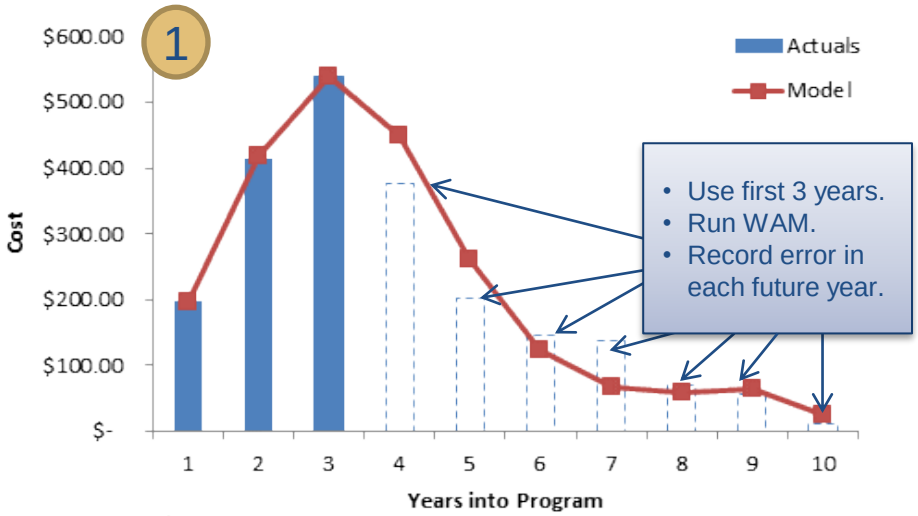
$$\min \sum_i \left(E(t_i) - \hat{E}(t_i)\right)^2$$

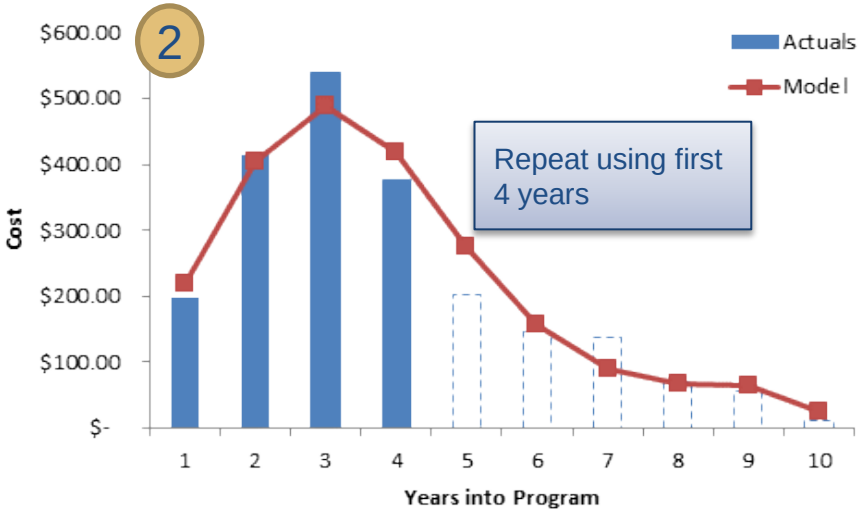
- Forecasting:
  - Apply α, β to project expenditures in remaining years
  - Convert to TY$ and compare to funding plans

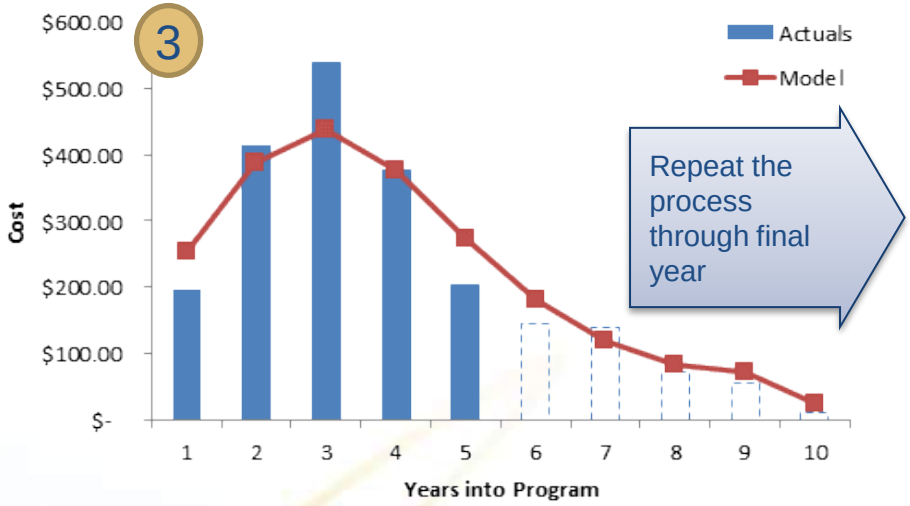
Measuring the Accuracy of WAM

- Gather and normalize historical phased expenditure data from 38 completed contracts
- Use WAM to generate estimates of “future” time phased program expenditures starting from progressively further points in each program
- Compare the WAM predicted time phased expenditures to the actual time phased expenditures and measure the error of the prediction
- Create a model to characterize WAM accuracy
- Compare the accuracy of WAM to the accuracy of the baseline phasing model

Generating Error Measurements







$$\text{Error, } \varepsilon_i = \frac{E(t_i) - \hat{E}(t_i)}{\hat{E}(t_i)}$$

Each program generated 20-40 measurements for a total of 1328 "Error Points"

Results for One Contract

Time of Estimate →

Time of Actuals ↓	20%	30%	40%	50%	60%	70%	80%	90%	100%
20%	N/A	-34%	-34%	1%	59%	86%	86%	223%	92%
30%	N/A	N/A	-11%					108	53%
40%	N/A	N/A	N/A	-1%	-12%	-27%	-20%		
50%	N/A			/A	-11%	-27%	-21%		0%
60%	N/A			/A	N/A	-28%	-25%		20%
70%	N/A			/A	N/A	N/A	-31%	39%	-2%
80%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	19%	-21%
90%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0%
100%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

WAM Percent Error

Sunk Costs: N/A

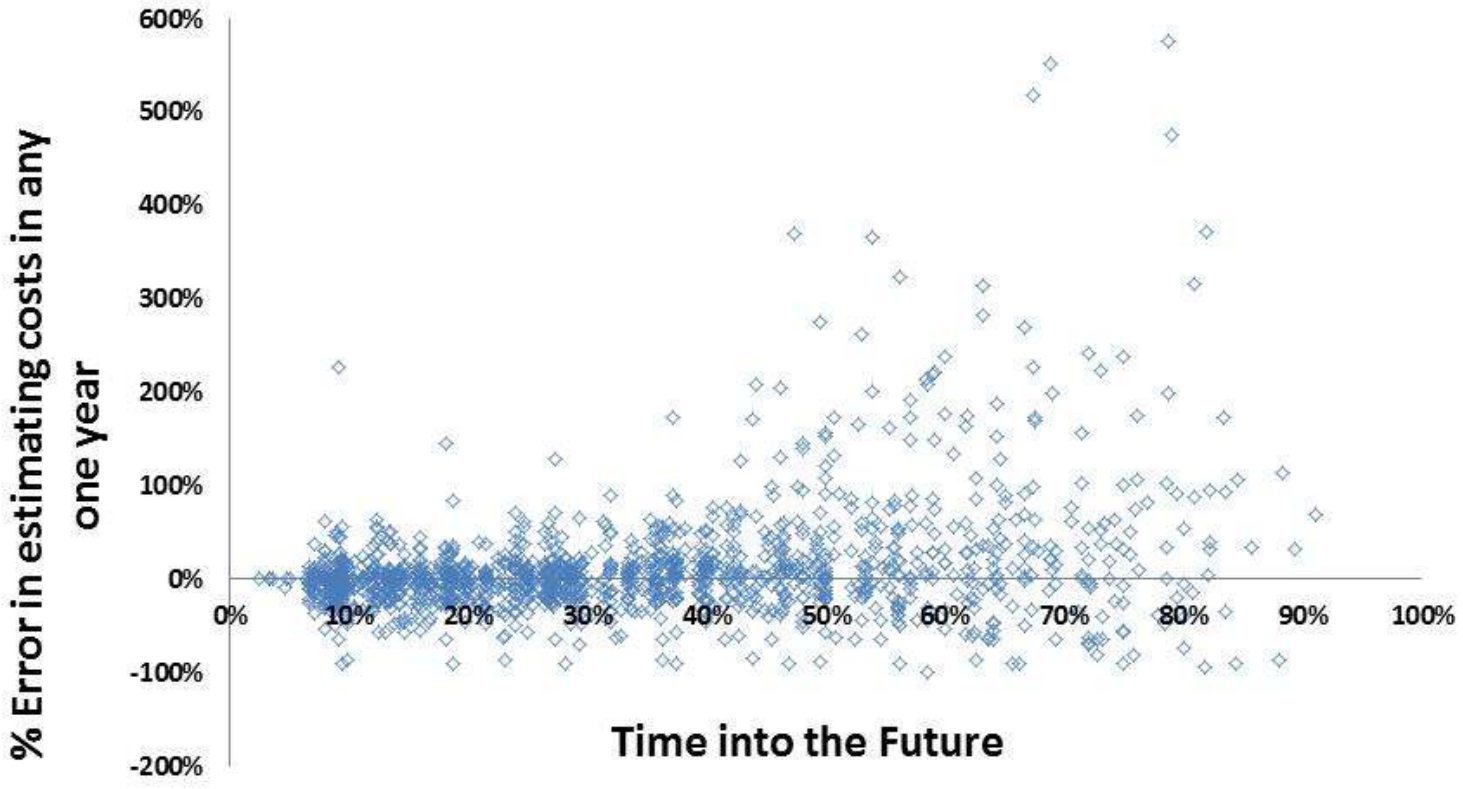
36 "Error Points"

- Time of Actuals: Data at and before this time is used for WAM best fit
- Time of Estimate: Time in contract for which WAM is estimating the expenditure level

Each program generated 20-40 measurements for a total of 1328 "Error Points"

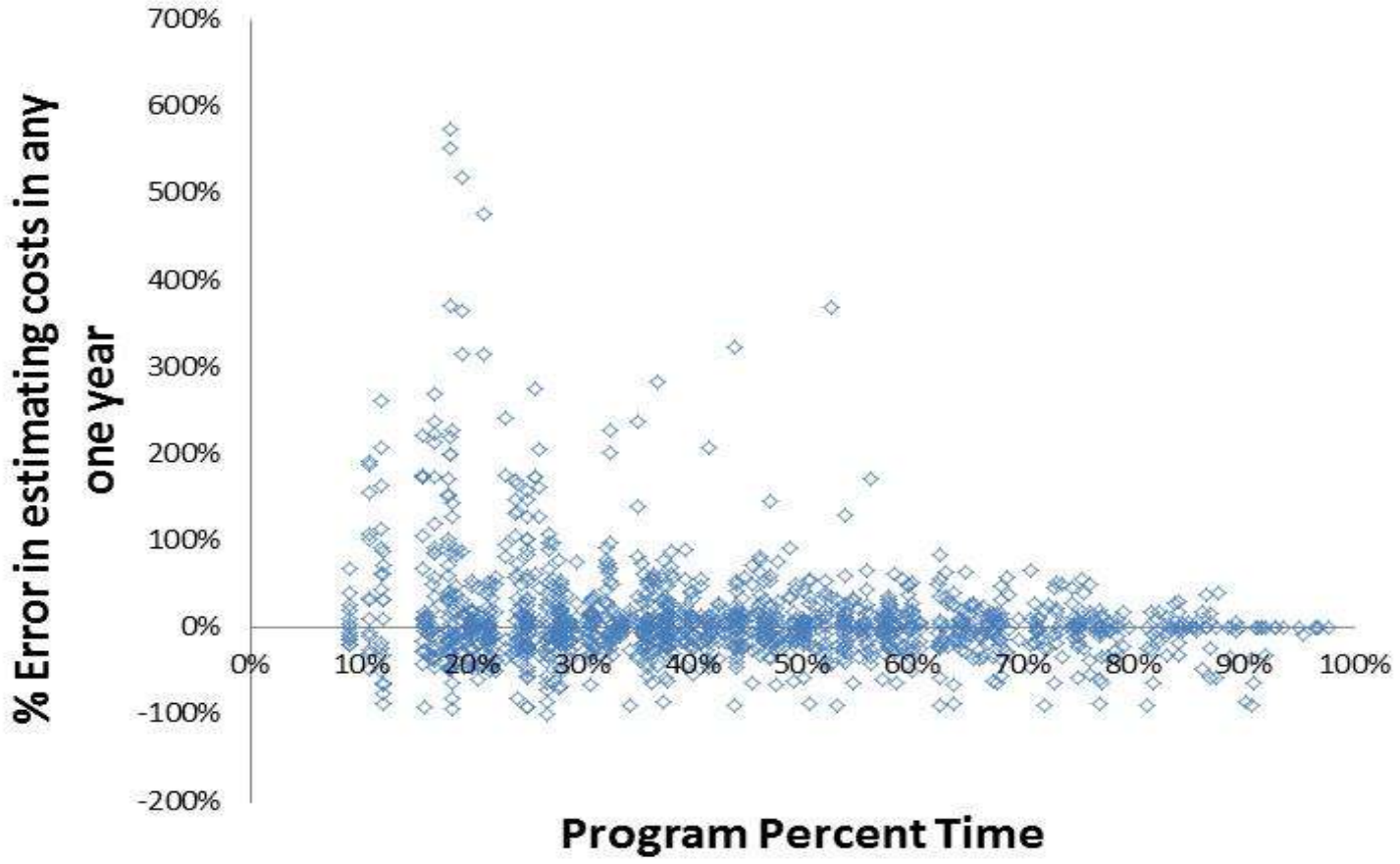
Error Source 1

Error increases when projecting farther into the future



Error Source 2

Error is greater when there are fewer years of data



WAM Error Model

- All 1328 points used in OLS regression to estimate absolute % error

$$|\%Error| = 0.052 - 0.123 * T_A + 0.968 * T_F$$

$$T_A = \frac{\text{Time of last actual}}{\text{Total time}}$$

$$T_F = \frac{\text{Time into future}}{\text{Total time}}$$

Error increases with T_F *and* decreases with T_A

- Both T_A and T_F are statistically significantly correlated with $|\%Error|$

ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	58.60	29.30	129.40	0.000
Residual	1325	300.01	0.23		
Total	1327	358.61			

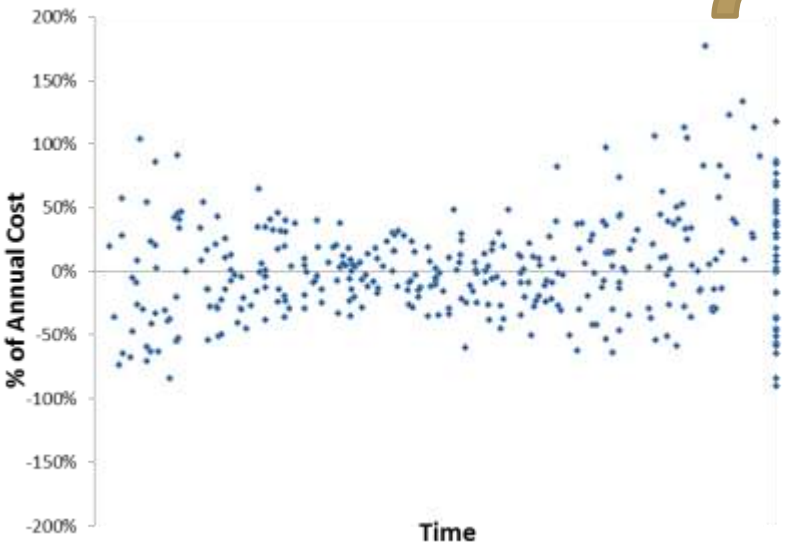
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.05	0.05	1.02	0.308
TA	-0.12	0.07	-1.67	0.094
TF	0.97	0.07	12.92	0.000

- Since the coefficient modifying T_F is much larger and the variable ranges are similar, T_F has much more impact on WAM error

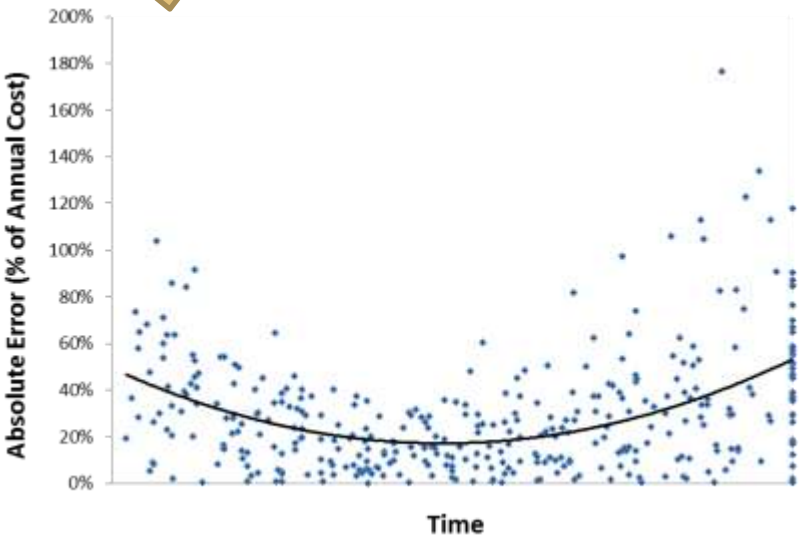
WAM Error is better when (1) the contract is father along, and (2) projecting near-term spending.

Absolute Error of Baseline Phasing Model

BASELINE MODEL:
Annual-Cost Error

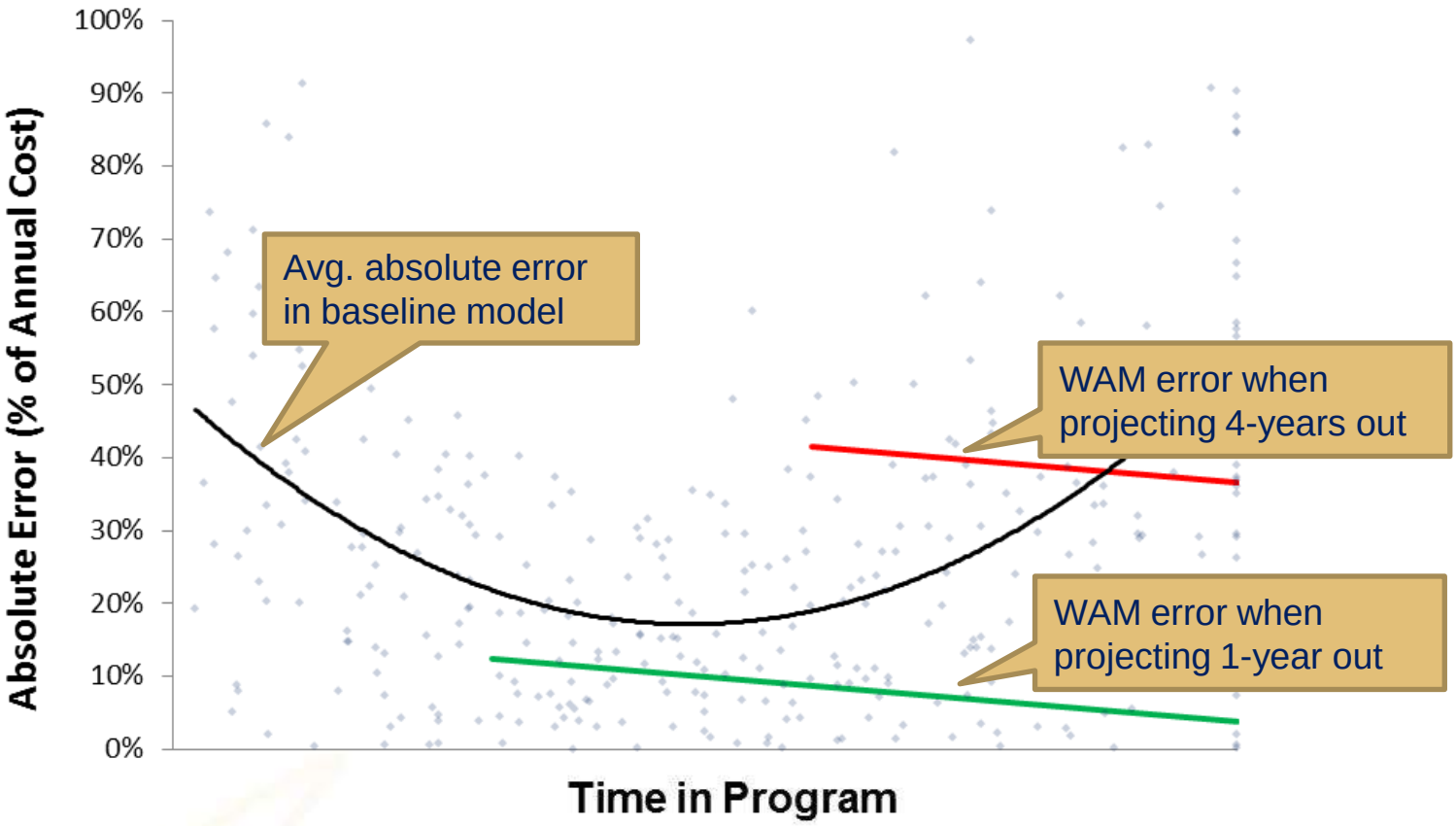


BASELINE MODEL:
Absolute Error



Absolute Error of Baseline Model is lower in the middle of a program when expenditures are high

Comparison to Baseline Model



WAM is a lot better at estimating program budgets in the near term and not as good at estimating far into the future

Weibull Analysis Tool (WAT)

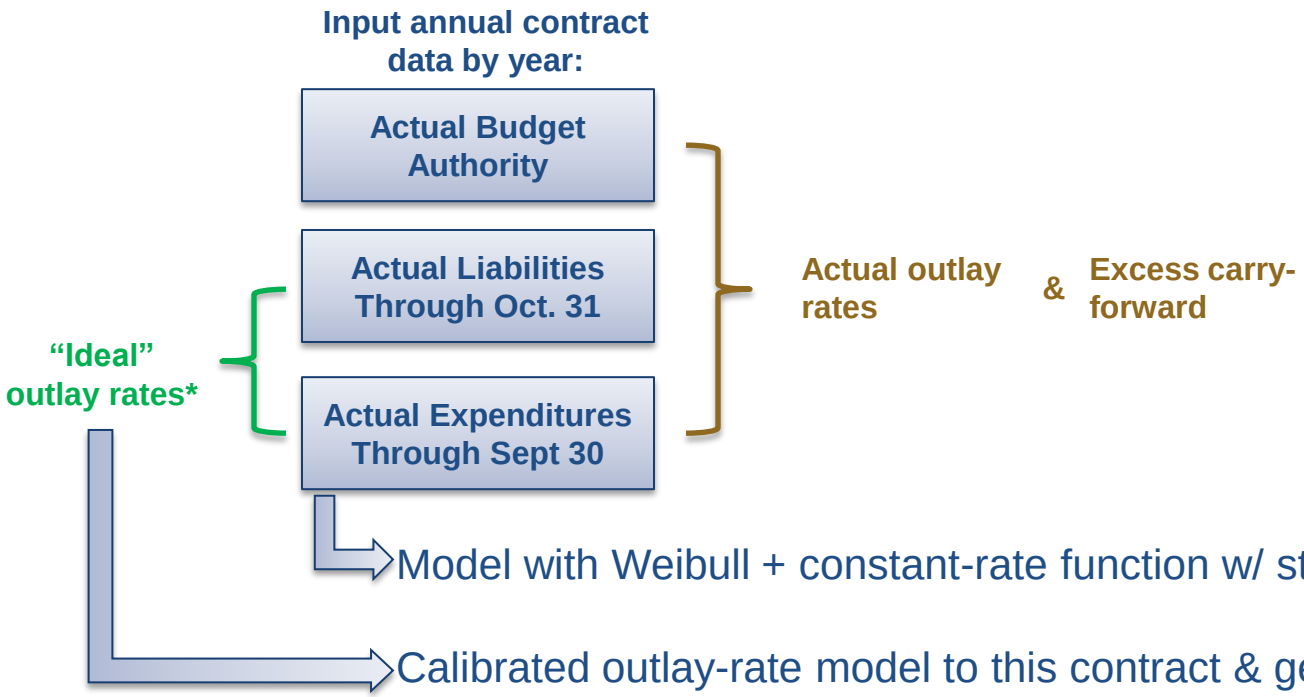
Implements WAM for NRO Estimators

WAT Goals

- Tool for NRO estimators
  - Apply WAM as repeatable part of estimating process
  - Excel-based, easy to integrate and modify
- Accepts and forecasts all relevant contract data
  - Expenditures
  - Government liability
  - Budget authority
  - Carry-forward
  - Actual program outlay rates
- Compare WAM result to:
  - Existing budget line
  - Program plan (CFSR)
  - Baseline phasing model

Are they within WAM error bounds?
Is there excess margin in any year?

Overview of WAT Mechanics



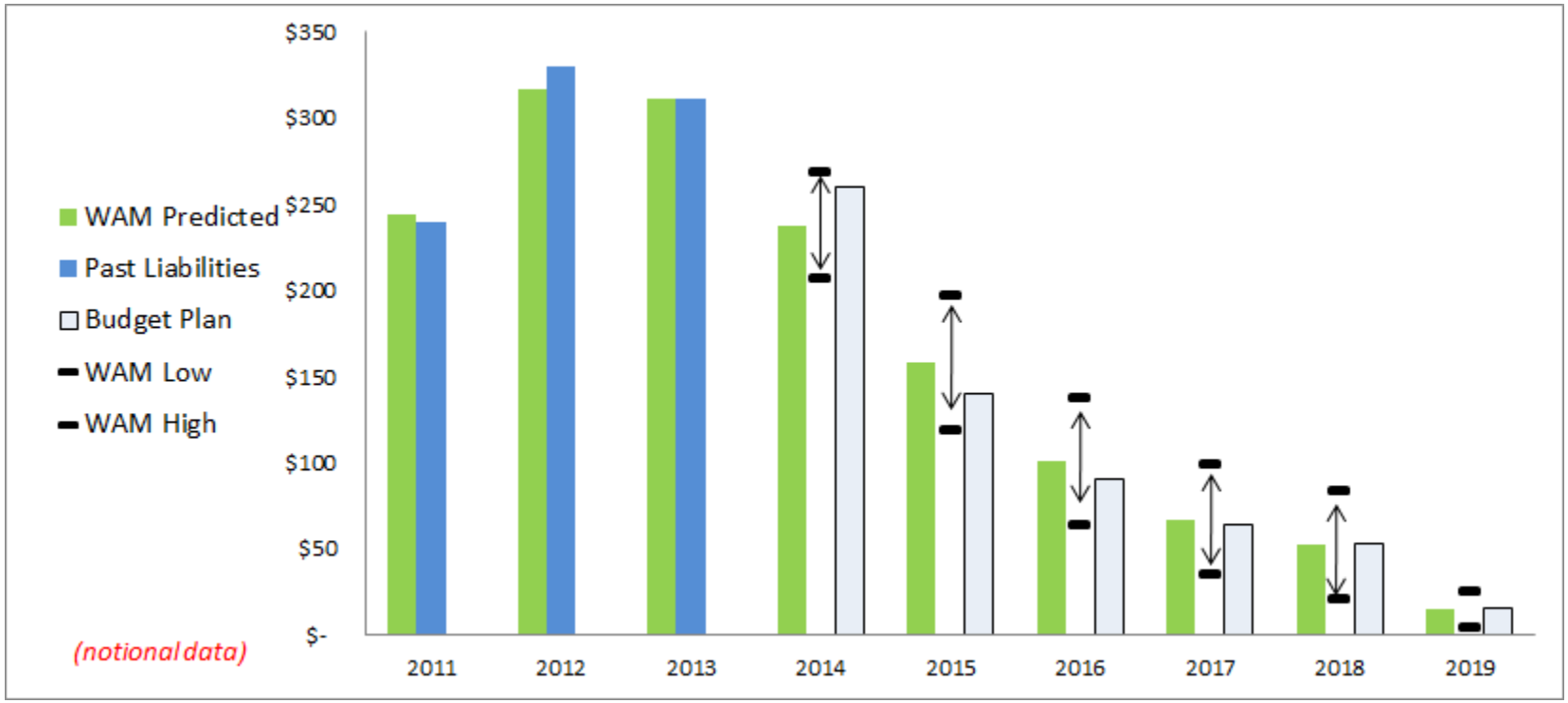
Model with Weibull + constant-rate function w/ strawman α, β

Calibrated outlay-rate model to this contract & generate liability curve

- Solve for α, β to fit liability curve through current year
  - Apply constraints, including budget already programmed
- Project future liabilities
  1. Hold excess carry-forward as margin
  2. Assume excess carry-forward eliminated in next budget year

*Based on budget authority needed to cover liabilities through 1 additional month, per NRO policy CBP 20, 30 June 2010

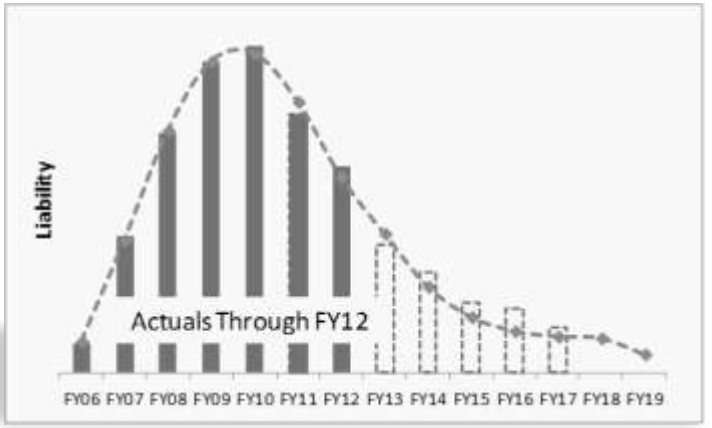
Application Example



Notional Data	2011	2012	2013	2014	2015	2016	2017	2018	2019
WAM Output (TY$)	$ 244	$ 316	$ 311	$ 238	$ 158	$ 101	$ 67	$ 53	$ 15
Past and Projected (TY$)	$ 239	$ 330	$ 311	$ 260	$ 140	$ 90	$ 64	$ 53	$ 15
Error vs. WAM (% annual)	-2%	4%	0%	9%	-12%	-11%	-5%	0%	0%
WAM Error model (% annual)	N/A	N/A	N/A	13%	25%	36%	48%	60%	71%
Baseline Phasing Model (TY$)	$ 239	$ 310	$ 311	$ 241	$ 162	$ 104	$ 69	$ 54	$ 15
Δ from Phasing Model (% cum)	0%	1%	1%	3%	1%	0%	0%	0%	0%

Summary

- WAM is a useful addition to NRO's estimating toolkit
- Serves as alternative to baseline phasing model
  - More accurate in near years
  - Calibrated to program-specific outlay patterns
- WAT integrates analysis of several key metrics
  - Expenditures
  - Outlay rates
  - Government liability
  - Budget authority
  - Carry forward



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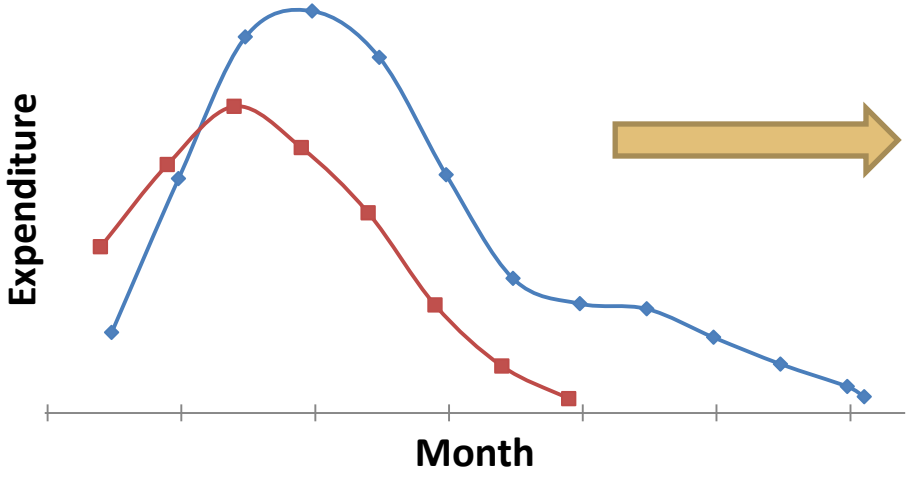
Backup

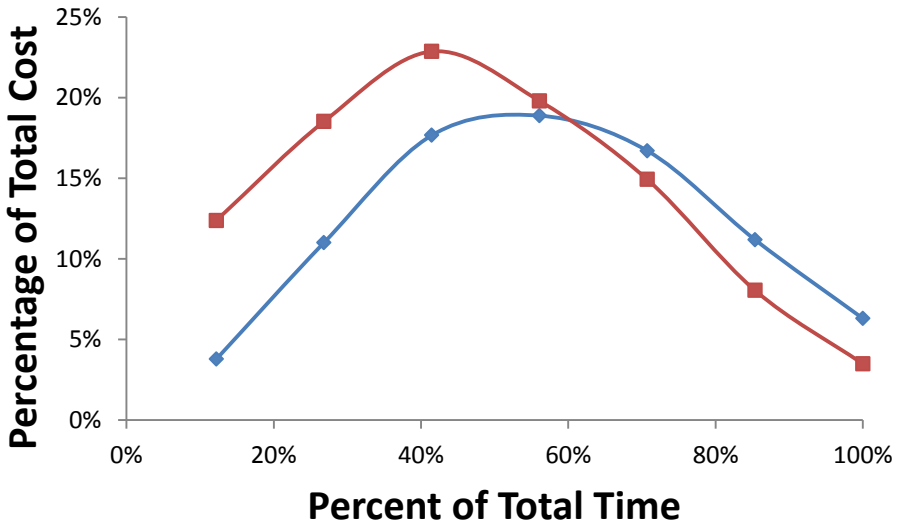
Normalizing the Data

Cost and time are normalized so profiles can be compared

Not Normalized



Normalized



Longer More Expensive Program

Shorter and Cheaper Program