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Weibull Analysis Method

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Topics

- Analytical Basis
- Accuracy
- Application

Weibull Analysis Method (WAM)

- Uses a program's actual history to estimate future budgets
  - Expenditures
  - Outlay rates
  - Government liability
- Improves accuracy over NRO's baseline parametric phasing model
  - For programs already underway
  - Especially in the near term
    - 2-years out (budget year)
    - FYDP
- Quantifies and reports error bounds based on historical data
  - Annual error vs. historical data
  - Departure from baseline model
- Builds on LMI[1] and CNA[2] research

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[1] Dukovich, John et al., "The Rayleigh Analyzer." Logistics Management Institute AT902C1. October, 1999.
[2] Davis, Dan et al. "Using the Rayleigh Model to Assess Future Acquisition Contract Performance and Overall Contract Risk." Center for Naval Analysis CRM D0019289.A2, January 2009.

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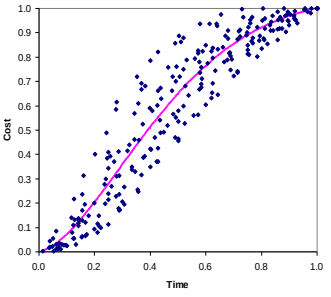
NRO's Baseline Parametric Phasing Model[3]



Weibull plus a constant-rate term
38 NRO & DoD Programs
387 time-cost pooled data points

$$E(t) = d\left[Rt + 1 - e^{-\alpha t^{\beta}}\right]$$

$$d = \frac{\text{total cost}}{R + 1\text{-}e^{-\alpha}}$$

$$0 \le t \le 1.0$$

$$R = .002945 \cdot \text{duration (mos.)}$$

$$\alpha = 0.10 + \sum X_i \cdot \text{driver}_i$$

$$\beta = 1.539 + \sum Y_i \cdot \text{driver}_i$$

- Adjusts front/back-loading based on "phasing drivers"
- Starting point for all space-segment estimates
- Phases expenditures, converts to budget authority

Driver	Coefficient (X)
GFE (1,0)	1.84E+00
% Subs	2.73E-02
BY07$M	9.57E-04
Duration (mos)	2.79E-02

Driver	Coefficient (Y)
Competitive (1,0)	1.71E-01
GFE (1,0)	3.62E-01
% Subs	4.47E-03
BY07$M	7.03E-05
Duration (mos)	-1.62E-03

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[3] Burgess, Erik. "Modeling R&D Budget Profiles," presented at SCEA/ISPA Joint Annual Conference, Orlando, FL. June 2012.

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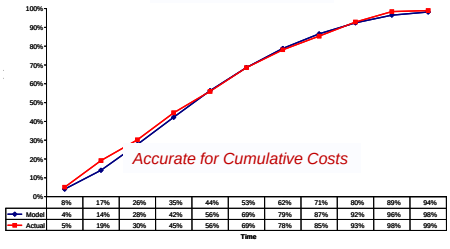
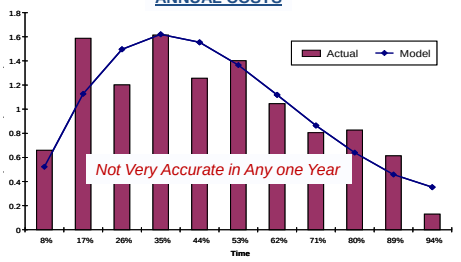
Baseline Model: Not Very Accurate in Any one Year

CUMULATIVE COSTS



	8%	17%	26%	35%	44%	53%	62%	71%	80%	89%	94%
Model	4%	14%	28%	42%	56%	69%	79%	87%	92%	96%	98%
Actual	5%	19%	30%	45%	56%	69%	78%	85%	93%	98%	99%

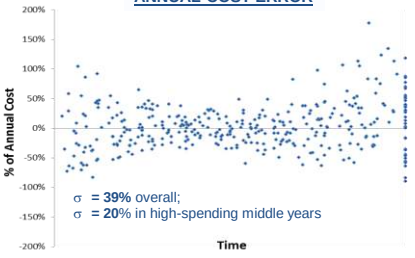
ANNUAL COSTS



CUMULATIVE-COST ERROR

σ = **6%** overall;
σ = **9**% through critical early years

ANNUAL-COST ERROR



σ = **39%** overall;
σ = **20**% in high-spending middle years

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Motivation for WAM

Baseline model establishes solid historical reference

- Cumulative accuracy through early years is quantified
- Powerful tool to link budget profile to schedule

But …

- Mid and late-program assessments now occur every year
  - Comparing government estimate at complete (GEAC) to program-office plan
  - Search for margin
  - Re-phasing the ICE
- Better method needed for evaluating annual budgets
  - Baseline model not very accurate for annual costs, especially in later years
  - Unclear how to apply baseline model when prior-year actuals are different
  - Need a method based on actuals, not plans

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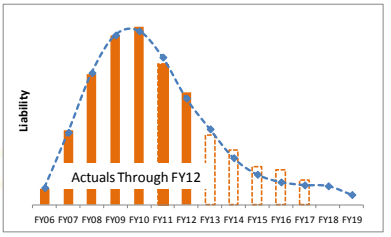
WAM: Analytical Basis

- Functional form: Weibull plus constant-rate term
  - Same as baseline phasing model
  - Empirical and theoretical basis for satellite acquisitions[4]

$$E(t) = d\left[\, Rt + 1 - e^{-\alpha t^{\beta}} \,\right]$$

$$d = \frac{\text{total cost}}{R + 1\text{-}e^{-\alpha}}, 0 \le t \le 1$$

- Use actual program performance to estimate Weibull parameters



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[4] Burgess, Erik. "R&D Budget Profiles and Metrics". *Journal of Parametrics*. Volume VVX, Issue No. 1, Summer 2006.

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Basic WAM Process

- Input: Actual expenditures for each year to date, BY$
- Constraints:
  - Total cost in BY$ (set to match ICE)
  - Schedule (set to match ICE)
  - Cumulative expenditures to date
  - Constant-rate term from baseline model: *R* = .002945 * duration

Used for re-phasing existing ICE. Unconstrained version also can be run.

- Optimization:
  - For each year *i* of actual data: $E(t_i) = d\left[\, Rt_i + 1 - e^{-\alpha t_i^{\beta}} \,\right], d = \frac{\text{total cost}}{R + 1\text{-}e^{-\alpha}}$
  - Excel Solver© estimates Weibull parameters α, β by minimization:

$$\min \sum_i \left(E(t_i) - \hat{E}(t_i) \right)^2$$

- Forecasting:
  - Apply α, β to project expenditures in remaining years
  - Convert to TY$ and compare to funding plans

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Measuring the Accuracy of WAM

- Gather and normalize historical phased expenditure data from 38 completed contracts
- Use WAM to generate estimates of "future" time phased program expenditures starting from progressively further points in each program
- Compare the WAM predicted time phased expenditures to the actual time phased expenditures and measure the error of the prediction
- Create a model to characterize WAM accuracy
- Compare the accuracy of WAM to the accuracy of the baseline phasing model

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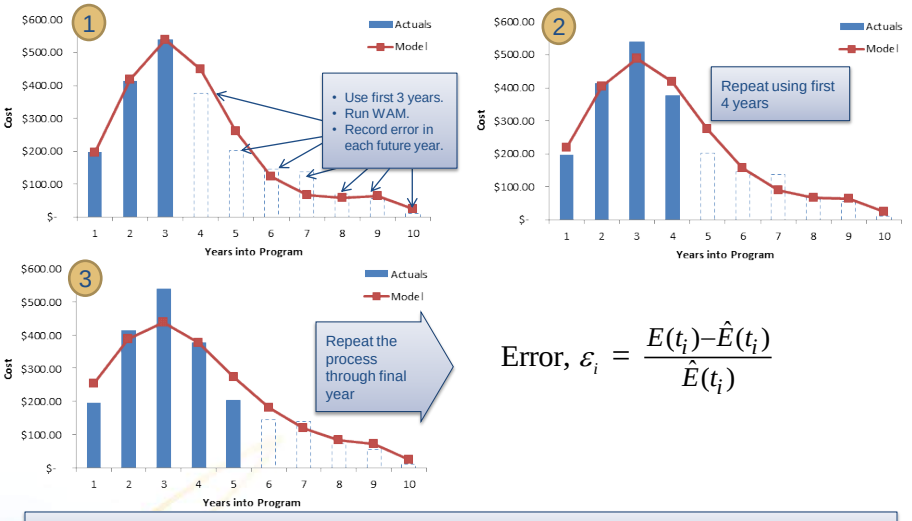
Generating Error Measurements



$$\text{Error, } \varepsilon_i = \frac{E(t_i) - \hat{E}(t_i)}{\hat{E}(t_i)}$$

Each program generated 20-40 measurements for a total of 1328 "Error Points"

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Results for One Contract

Time of Estimate

Time of Actuals	20%	30%	40%	50%	60%	70%	80%	90%	100%
20%	N/A	-34%	-34%	1%	59%	86%	86%	223%	92%
30%	N/A	N/A	-11%				%	108	52%
40%	N/A	N/A	N/A	-1%	-12%	-27%	-20%		
50%	N/A			/A	-11%	-27%	-21%		0%
60%	N/A			/A	N/A	-28%	-25%		20%
70%	N/A			/A	N/A	N/A	-31%	39%	-2%
80%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	19%	-21%
90%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0%
100%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

WAM Percent Error

36 "Error Points"

Sunk Costs: N/A

- Time of Actuals: Data at and before this time is used for WAM best fit
- Time of Estimate: Time in contract for which WAM is estimating the expenditure level

Each program generated 20-40 measurements for a total of 1328 "Error Points"

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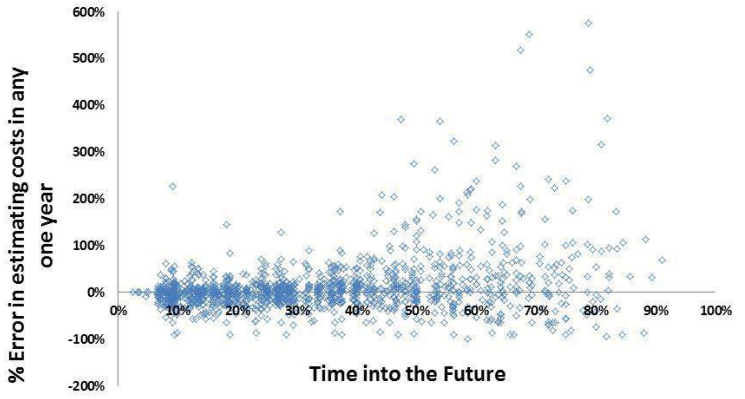
Error Source 1

Error increases when projecting farther into the future



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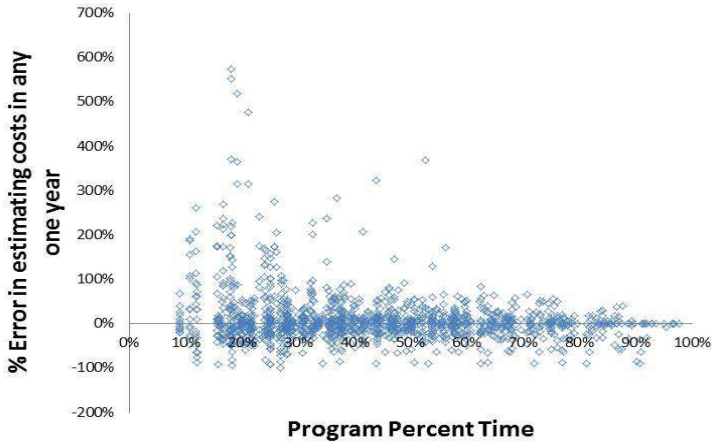
Error Source 2

Error is greater when there are fewer years of data



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WAM Error Model

- All 1328 points used in OLS regression to estimate absolute % error

$$|\%Error| = 0.052 - 0.123 * T_A + 0.968 * T_F$$

$$T_A = \frac{\text{Time of last actual}}{\text{Total time}}$$

$$T_F = \frac{\text{Time into future}}{\text{Total time}}$$

Error increases with T_F *and* decreases with T_A

- Both T_A and T_F are statistically significantly correlated with $|\%Error|$

ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	58.60	29.30	129.40	0.000
Residual	1325	300.01	0.23		
Total	1327	358.61			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.05	0.05	1.02	0.308
TA	-0.12	0.07	-1.67	0.094
TF	0.97	0.07	12.92	0.000

- Since the coefficient modifying T_F is much larger and the variable ranges are similar, T_F has much more impact on WAM error

WAM Error is better when (1) the contract is father along, and (2) projecting near-term spending.

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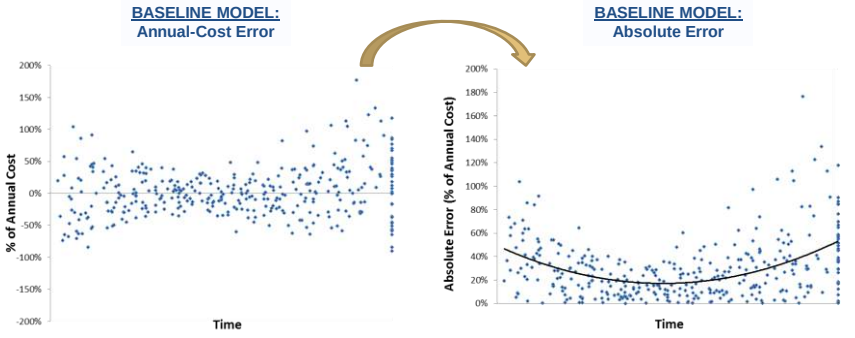
Absolute Error of Baseline Phasing Model



Absolute Error of Baseline Model is lower in the middle of a program when expenditures are high

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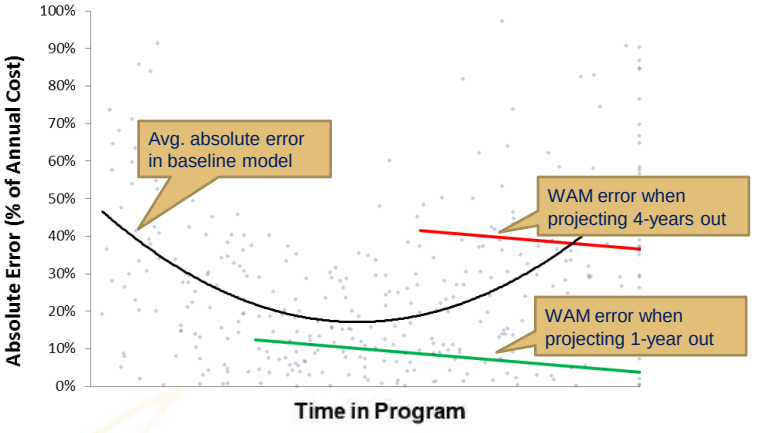
Comparison to Baseline Model



WAM is a lot better at estimating program budgets in the near term and not as good at estimating far into the future

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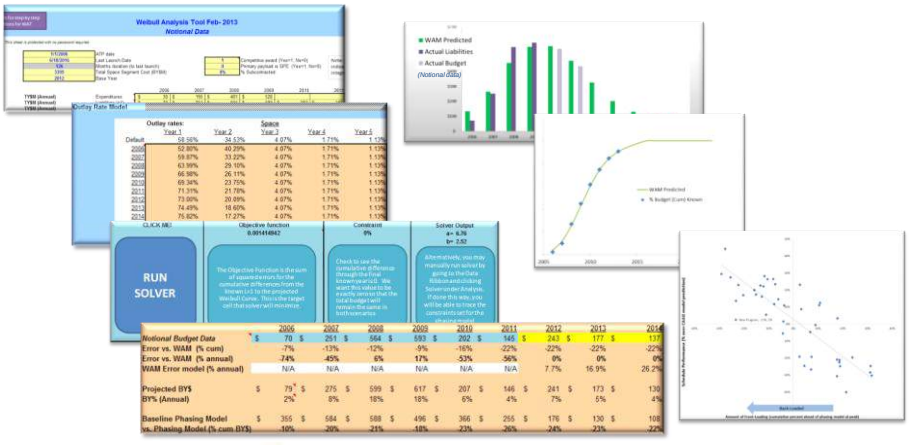
Weibull Analysis Tool (WAT)



Implements WAM for NRO Estimators

WAT Goals

- Tool for NRO estimators
  - Apply WAM as repeatable part of estimating process
  - Excel-based, easy to integrate and modify
- Accepts and forecasts all relevant contract data
  - Expenditures
  - Government liability
  - Budget authority
  - Carry-forward
  - Actual program outlay rates
- Compare WAM result to:
  - Existing budget line
  - Program plan (CFSR)
  - Baseline phasing model

Are they within WAM error bounds?
Is there excess margin in any year?

Overview of WAT Mechanics

Input annual contract data by year:

- Actual Budget Authority
- Actual Liabilities Through Oct. 31
- Actual Expenditures Through Sept 30

"Ideal" outlay rates*

Actual outlay rates & Excess carry-forward

Model with Weibull + constant-rate function w/ strawman α, β

Calibrated outlay-rate model to this contract & generate liability curve

- Solve for α, β to fit liability curve through current year
  - Apply constraints, including budget already programmed
- Project future liabilities
  1. Hold excess carry-forward as margin
  2. Assume excess carry-forward eliminated in next budget year

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*Based on budget authority needed to cover liabilities through 1 additional month, per NRO policy CBP 20, 30 June 2010

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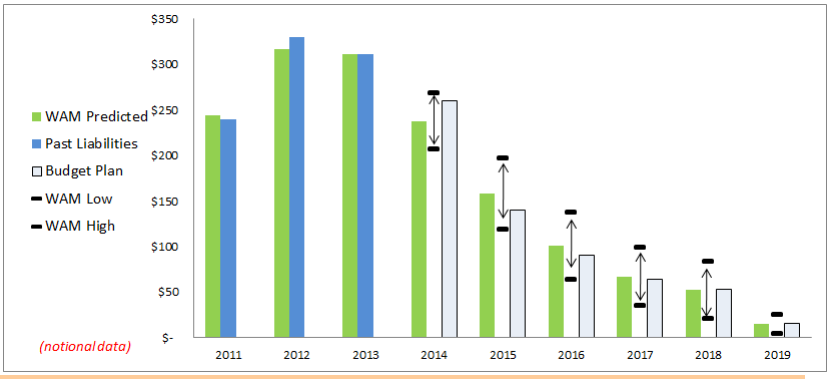
Application Example



Notional Data	2011	2012	2013	2014	2015	2016	2017	2018	2019
WAM Output (TY$)	$ 244	$ 316	$ 311	$ 238	$ 158	$ 101	$ 67	$ 53	$ 15
Past and Projected (TY$)	$ 239	$ 330	$ 311	$ 260	$ 140	$ 90	$ 64	$ 53	$ 15
Error vs. WAM (% annual)	-2%	4%	0%	9%	-12%	-11%	-5%	0%	0%
WAM Error model (% annual)	N/A	N/A	N/A	13%	25%	36%	48%	60%	71%
Baseline Phasing Model (TY$)	$ 239	$ 310	$ 311	$ 241	$ 162	$ 104	$ 69	$ 54	$ 15
Δ from Phasing Model (% cum)	0%	1%	1%	3%	1%	0%	0%	0%	0%

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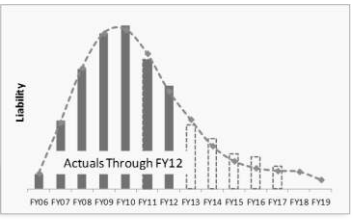
Summary

- WAM is a useful addition to NRO's estimating toolkit
- Serves as alternative to baseline phasing model
  - More accurate in near years
  - Calibrated to program-specific outlay patterns
- WAT integrates analysis of several key metrics
  - Expenditures
  - Outlay rates
  - Government liability
  - Budget authority
  - Carry forward



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Backup

Normalizing the Data

Cost and time are normalized so profiles can be compared